Transport Computations for the North Pacific Ocean
1955

by
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Introduction

Sverdrup (1947), Stommel (1948) and Munk (1950) have shown that the total steady-state transport of mass in the ocean depends primarily on the curl of the wind stress components acting on the surface of the ocean. Computations based on this theoretical result have yielded transports which agree quite well with the distribution and magnitude of transport inferred from oceanographic observations. In the present undertaking, the steady-state model of mass transport has been applied to extensive calculation of transports in the North Pacific Ocean. These calculations were undertaken with two objectives in mind. First, to find out how long an averaging period is necessary to approach steady-state conditions and second, to gain some idea of the variability, both with latitude and time, of the transport that is imposed on the ocean by the atmosphere.

Estimates of surface wind stress

Direct estimates of surface stress induced by winds are not available. These have been obtained by computing geostrophic winds from the mean sea-level atmospheric pressure over the North Pacific. The method used to estimate surface stresses is related to that used by Montgomery (1935). Monthly means of sea-level atmospheric pressure are issued by the Extended Forecast Section of the U.S. Weather Bureau. These are obtained by averaging pressures read twice daily from synoptic charts at a regular set of grid points covering the northern hemisphere. The pressure means are interpolated to a standard grid consisting of alternate points of a 5-degree rectangular grid in latitude and longitude.

Pressures from the region 10°N to 65°N and 115°E to 105°W were read from the charts for the transport computations.

The computations at a grid point are made in terms of the mean pressure at that point and the six surrounding grid points. The geostrophic velocity vector is computed first. It is rotated 15° to the left of the downwind direction and reduced to 70% of its original magnitude to represent surface wind. The stress vector is assumed to be in the direction of the surface wind and to be proportional to the square of the wind speed. The stress law used is

$$|T_s| = \rho_a v^2 v$$

where \(\rho_a\) is air density taken as 1.22 x 10^{-3} gm/cm^3, \(v^2\) is a nondimensional drag coefficient chosen to be 2.6 x 10^{-3} and \(v\) is wind speed in centimetres per second.
Using the equations for geostrophic velocity, the transformation for surface velocity and the stress law, the components of Ekman and total transport are expressed in terms of the pressures at the seven points forming the computational grid pattern. The details of the computations and a description of the computer program have been given by Fofonoff and Froese (1960).

It must be recognized that computations of wind stress from the mean pressure distribution are subject to considerable uncertainty. The surface stress is not a unique function of the pressure distribution at sea-level but depends also on the variation of geostrophic winds with altitude. The use of monthly means of pressure introduces further uncertainties. If the "square" law for stress is assumed, the stress computed from the mean pressure will differ from the mean stress computed from daily pressure means. The difference between the two means will depend on the amplitude of the fluctuations of wind velocity. To illustrate this point, it can be assumed that stress magnitude can be expressed as a pressure difference between grid points, i.e.

\[ |T_{s}| \propto u^2 \propto (\Delta P)^2. \]

The mean stress magnitude computed from daily pressure differences will be given by

\[ \frac{1}{N} \sum_{i} |T_{s_i}| = \frac{|T_{s}|}{N} \propto (\Delta P)^2, \]

whereas the stress from the monthly mean of pressure is

\[ |T_{s_m}| \propto (\Delta \bar{P})^2. \]

Hence, the difference between the two means is

\[ |T_{s}| - |T_{s_m}| \propto (\Delta P)^2 - (\Delta \bar{P})^2 = \text{Variance} (\Delta P). \]

An estimate of the error introduced by using monthly pressure means can be obtained by calculating the statistical variance of pressure differences between grid points.

Another estimate of the error introduced by using means of pressure can be obtained by comparing annual means of transport with the transport computed from the annual mean of sea-level pressure. These comparisons will be reported separately.
Interpretation of the transport charts

The calculations for each month are carried out independently of the other months. Thus, the transports for each month must be interpreted as the limiting transport that would result if the pressure distribution for that month were to persist without change for an indefinitely long period of time. No account of inertia of the ocean is taken in going from one monthly mean to another. Thus, the computed transports can only indicate tendencies of the ocean circulation. If the computed transport for a given month is in excess of actual transport, the ocean circulation could be expected to increase at a rate depending on the difference between the computed and actual transport and on internal inertial response characteristics. The computed transports can be regarded as indices indicating the relative range and frequency of variations applied to the ocean by the atmosphere.

Long-term averages of transport computed from the pressure distribution should converge to the mean transport observed in the ocean within the limits of accuracy of the method. It should be remembered that the transports are very sensitive to the proportionality factors used in relating geostrophic to surface wind and surface wind to surface stress. Numerical equivalence of computed and observed transports is not anticipated.

Acknowledgement

Charts of mean sea-level atmospheric pressure and information on the methods used for their compilation were made available through the courtesy of Mr. Jerome Namais, Chief of the Extended Forecast Section and Mr. Roy Fox, Director of the National Weather Records Centre of the U.S. Weather Bureau. The computer program for obtaining transport components was constructed by Dr. Charlotte Froese of the Department of Mathematics, University of B. C. The charts were prepared from the computer tapes by Miss Mary Cairns. Contouring and preparation of the charts for publication was carried out by Messrs. Norman Binda, J. Allan Coombs, Jack Gow and Wayne Harling. The assistance of all of these individuals is gratefully acknowledged.

References


Description of Transport Charts

Section I - Atmospheric pressure

Mean monthly sea-level atmospheric pressure charts are given in the first section. The pressures are obtained from the northern hemisphere charts of mean sea-level pressure issued by the Extended Forecast Section of the U.S. Weather Bureau. The pressures are read for 144 grid points covering the North Pacific Ocean from 10°N to 65°N and 115°E to 105°W and are given as pressure less 1000 millibars in units of 1/10 millibar. A chart is given for each month, and for the annual mean computed as the arithmetical average of the twelve monthly charts.

Section II - Meridional component of Ekman transport

The Ekman transport is induced directly by the action of the surface wind stress. In the steady state, it is directed at right angles to the direction of the wind stress and is equal in magnitude to the stress divided by the coriolis parameter. The Ekman transport is assumed to be confined to the upper layers of the ocean, and, hence, is an indication of the movement of the surface waters.

The meridional component of Ekman transport is given by

\[ V_e = - \frac{\tau_x}{f} \]

where \( \tau_x \) is the zonal component of wind stress in dynes per square centimetre and \( f \) is the coriolis parameter in radians per second. The component is given in the charts in units of 10 metric tons* per second per kilometre (decimal point following the last digit). A zonal component of wind stress of 1 dyne/cm² at a latitude for which \( f = 10^{-4} \) sec⁻¹ gives rise to a meridional transport of 1000 metric tons per second per kilometer (or 1 metric ton per second per metre) and would be recorded on the charts as 0100. The charts can also be used to estimate the zonal component of wind stress by the equation

\[ \tau_x = - f V_e \]

Values of \( f \) are given in Table I.

*1 metric ton = 10⁶ gms
Section III - Zonal component of Ekman transport

The zonal component of Ekman transport is obtained from the meridional component of wind stress $\tau_\phi$ by the formula

$$U_E = \frac{\tau_\phi}{f}$$

and is given in the charts in units of 10 metric tons per second per kilometre. (cf: Section II).

The meridional component of wind stress can be obtained from these charts using the formula

$$\tau_\phi = f U_E$$

Values of $f$ are given in Table I.

Section IV - Meridional component of total mass transport

The total meridional transport of mass across a unit length of a latitude circle is given by the formula

$$V = \frac{C\mu r_\lambda}{\beta}$$

where $\beta$ is the rate of variation of the coriolis parameter along a meridian and is equal to $\frac{df}{R df/d\phi}$. As the total transport satisfies the continuity equation, a transport function $\psi$ can be introduced such that its derivatives yield the components of total transport

$$U = \frac{\psi}{R \cos \psi}$$

$$V = -\frac{1}{R \cos \psi} \frac{\partial \psi}{\partial \lambda} = \frac{C\mu r_\lambda}{\beta}$$

Values of $\frac{\psi}{R \cos \psi} \frac{\partial \psi}{\partial \lambda}$ are given in this section in units of 100 metric tons per second per kilometre (decimal following the last digit). Thus, a positive value indicates a total transport per unit width towards the equator.

The charts of $\frac{\psi}{R \cos \psi} \frac{\partial \psi}{\partial \lambda}$ can also be used to estimate the divergence of the Ekman transport from the relationship

$$\text{DIV} U_E = -\beta f \left( \frac{\sigma}{R \cos \psi} \frac{\partial \psi}{\partial \lambda} + V_e \right)$$

Values of $\beta / f$ are given in Table I.
Section V - Integrated total transport

The meridional component of total transport is integrated westward from the grid point closest to the North American continent at the latitude under consideration. The integration is carried out across the ocean to the western side, but no provision is made for calculating the western boundary currents. It may be assumed by continuity that the net flow across a latitude circle from shore to shore is zero. Under this assumption, the values of the integrated transport nearest the western shores of the ocean are indicative of the transport of the western boundary currents (Kuroshio, Oyashio and the Alaskan stream). For simplicity in programming, the integrating was carried across the Aleutian island chain. Thus, values of \( \Psi \) in the Bering Sea require subtraction of the transport function value at the island chain. Because of lack of pressure data in the southeastern Pacific, the integration at 15°N and 20°N is incomplete and gives only the transports in the western section of the ocean relative to interior grid points.

The charts of integrated transport are given in units of 50,000 metric tons per second* (decimal point following the last digit).

Section VI - Integrated geostrophic transport

The meridional component of total transport less the meridional component of Ekman transport gives the net mass of water transported by the geostrophic current across latitude circles. The geostrophic mass transport is proportional to the zonal component of the gradient of potential energy according to the equation

\[
- \frac{1}{R \cos \phi} \frac{\partial \Psi_g}{\partial \lambda} = \sqrt{\gamma} \nu_E = \frac{1}{R \cos \phi} \frac{\partial \gamma}{\partial \lambda}, \quad \Psi_g = \frac{\lambda_0 - \lambda}{f}
\]

where \( \lambda_0 \) is the longitude of the eastern boundary grid point.

The integration is carried out using zero as the boundary condition for \( \Psi_g \) at the grid point nearest the eastern coast. Thus, although the east-west changes of potential energy are given by the geostrophic transport function, the north-south components are not correctly represented. The difference between values of the total transport and the geostrophic transport functions at any grid point represents the contribution of the meridional Ekman transport to the integrated total transport eastward of the point.

The geostrophic transport is given in units of 50,000 metric tons per second.

*A division by two was inadvertently left out of the integration in the computer program.
Table I

Values of $f$, $\beta$, and $\beta/f$ at standard latitudes based on a mean earth radius of 6371.22 km and angular velocity of $0.72921 \times 10^{-5}$ radians per second.

<table>
<thead>
<tr>
<th>Latitude Degrees</th>
<th>$10^4 f$ sec$^{-1}$</th>
<th>$10^{-13} \beta$ cm$^{-1}$ sec$^{-1}$</th>
<th>$10^9 \beta/f$ cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
<td>2.2891</td>
<td>$\infty$</td>
</tr>
<tr>
<td>5</td>
<td>0.1524</td>
<td>2.2765</td>
<td>14.9334</td>
</tr>
<tr>
<td>10</td>
<td>0.2533</td>
<td>2.2543</td>
<td>8.9014</td>
</tr>
<tr>
<td>15</td>
<td>0.3775</td>
<td>2.2111</td>
<td>5.8577</td>
</tr>
<tr>
<td>20</td>
<td>0.4988</td>
<td>2.1510</td>
<td>4.3124</td>
</tr>
<tr>
<td>25</td>
<td>0.6164</td>
<td>2.0746</td>
<td>3.3659</td>
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<td>2.7186</td>
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<td>1.8751</td>
<td>2.2415</td>
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<tr>
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<td>1.7535</td>
<td>1.8706</td>
</tr>
<tr>
<td>45</td>
<td>1.0313</td>
<td>1.6186</td>
<td>1.5696</td>
</tr>
<tr>
<td>50</td>
<td>1.1172</td>
<td>1.4714</td>
<td>1.3170</td>
</tr>
<tr>
<td>55</td>
<td>1.1947</td>
<td>1.3130</td>
<td>1.0990</td>
</tr>
<tr>
<td>60</td>
<td>1.2630</td>
<td>1.1445</td>
<td>0.9062</td>
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<td>65</td>
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<td>0.9674</td>
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<td>75</td>
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<td>0.4206</td>
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<td>80</td>
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<td>0.3975</td>
<td>0.2768</td>
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<td>85</td>
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<td>0.2393</td>
<td>0.1650</td>
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<tr>
<td>90</td>
<td>1.4584</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Example

The annual mean transport components for 1955 at the grid point 50°N and 150°W as shown in the charts are:

\[ V_E = -0.068 \]
\[ U_E = +0.034 \]
\[ \frac{\partial \theta}{R \cos \phi \partial x} = -0.027 \]
\[ \psi = +0.064 \]
\[ \psi_g = +0.077 \]

These values can be interpreted in various mass transport units as shown in the following Table:

<table>
<thead>
<tr>
<th>Transport Component</th>
<th>Metric tons per second</th>
<th>gms per second</th>
<th>Millions of metric tons per second</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_E )</td>
<td>-680</td>
<td>-0.68</td>
<td>6,800</td>
</tr>
<tr>
<td>( U_E )</td>
<td>+340</td>
<td>+0.34</td>
<td>3,400</td>
</tr>
<tr>
<td>( \frac{\partial \theta}{R \cos \phi \partial x} )</td>
<td>-2700</td>
<td>-2.7</td>
<td>27,000</td>
</tr>
<tr>
<td>( \psi )</td>
<td>-</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td>( \psi_g )</td>
<td>-</td>
<td>-</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Mean surface stress components

Mean surface stress components $\tau_\phi$, $\tau_\lambda$ can be estimated from the equations

$$\tau_\phi = fU_e = 1.12 \times 10^{-4} \text{ sec}^{-1} \times 3400 \text{ gms sec}^{-1} \text{ cm}^{-1} = 0.38 \text{ dynes/cm}^2$$

$$\tau_\lambda = -fV_e = 1.12 \times 10^{-4} \text{ sec}^{-1} \times (-6800 \text{ gms sec}^{-1} \text{ cm}^{-1}) = 0.76 \text{ dynes/cm}^2$$

Divergence of Ekman transport

The divergence of Ekman transport is equal to the vertical velocity $w_e$ at the bottom of the surface Ekman layer, i.e.

$$\varphi W_e = \frac{1}{R \cos \psi} \left( \frac{\partial U_e}{\partial \lambda} + \frac{\partial V_e \cos \psi}{\partial \phi} \right) = \frac{\text{curl } Z}{f} - \frac{\beta V_e}{f}$$

$$= -\frac{\beta}{f} \left( \frac{1}{R \cos \psi} \frac{\partial \psi}{\partial \lambda} + V_e \right)$$

$$= -1.32 \times 10^{-9} \text{ cm}^{-1} (-27,000 - 6800) \text{ gms sec}^{-1} \text{ cm}^{-1}$$

$$= +4.5 \times 10^{-5} \text{ (gm cm}^{-3}) \text{ (cm sec}^{-1})$$

The positive sign indicates an upward movement of water into the surface layer of about $4.5 \times 10^{-5} \text{ cm/sec}$ ($\sim 14$ metres/year).

Integrated meridional transport in the Ekman layer

The mean northward transport between 150°W and 130°W is given by $\psi$ and equals 3.2 million metric tons per second. The geostrophic transport $\psi_g$ over the same section is 3.8 million metric tons per second. The difference $(\psi - \psi_g)$ of -0.6 million metric tons per second represents a southward movement in the surface layer due to Ekman transport.
Section I

Atmospheric Pressure
I. Atmospheric Pressure

MAY 31 - JUN 29, 1955
I. Atmospheric Pressure

JUL 2 - JUL 31, 1955
I. Atmospheric Pressure

AUG 2 - AUG 31, 1955
I. Atmospheric Pressure

OCT 1 - OCT 30, 1955
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II. Meridional Component of Ekman Transport

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II. Meridional Component of Ekman Transport

JAN 29 - FEB 27, 1955
II. Meridional Component of Ekman Transport

MAR 1 - MAR 30, 1955
II. Meridional Component of Ekman Transport

MAR 29 - APR 27, 1955
II. Meridional Component of Ekman Transport

MAY 31 - JUN 29, 1955
II. Meridional Component of Ekman Transport

JUL 2 - JUL 31, 1955
II. Meridional Component of Ekman Transport

AUG 2 - AUG 31, 1955
II. Meridional Component of Ekman Transport

AUG 30 - SEP 26, 1955
II. Meridional Component of Ekman Transport

OCT 1 - OCT 30, 1955
II. Meridional Component of Ekman Transport

NOV 1 - NOV 30, 1955
II. Meridional Component of Ekman Transport

NOV 29 - DEC 28, 1955
II. Meridional Component of Ekman Transport

ANNUAL MEAN 1955
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Zonal Component of Ekman Transport
III. Zonal Component of Ekman Transport

JAN 1 - JAN 30, 1955
III. Zonal Component of Ekman Transport

MAR 1 - MAR 30, 1955
III. Zonal Component of Ekman Transport

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III. Zonal Component of Ekman Transport

APR 30 - MAY 29, 1955
III. Zonal Component of Ekman Transport

MAY 31 - JUN 29, 1955
III. Zonal Component of Ekman Transport

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AUG 2 - AUG 31, 1955
III. Zonal Component of Ekman Transport

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III. Zonal Component of Ekman Transport

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III. Zonal Component of Ekman Transport

NOV 1 - NOV 30, 1955
III. Zonal Component of Ekman Transport

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III. Zonal Component of Ekman Transport

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Meridional Component of Total Transport
IV. Meridional Component of Total Transport  JAN 1 - JAN 30, 1955
IV. Meridional Component of Total Transport

JAN 29 - FEB 27, 1955
IV. Meridional Component of Total Transport

MAR 1 - MAR 30, 1955
IV. Meridional Component of Total Transport

MAR 29 - APR 27, 1955
IV. Meridional Component of Total Transport

APR 30 - MAY 29, 1955
IV. Meridional Component of Total Transport

MAY 31 - JUN 29, 1955
IV. Meridional Component of Total Transport

JUL 2 - JUL 31, 1955
IV. Meridional Component of Total Transport

AUG 2 - AUG 31, 1955
IV. Meridional Component of Total Transport

AUG 30 - SEP 28, 1955
IV. Meridional Component of Total Transport

OCT 1 - OCT 30, 1955
IV. Meridional Component of Total Transport

NOV 1 - NOV 30, 1955
IV. Meridional Component of Total Transport

NOV 29 - DEC 28, 1955
IV. Meridional Component of Total Transport

ANNUAL MEAN 1955
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Integrated Total Transport
V. Integrated Total Transport

APR 30 - MAY 29, 1955
V. Integrated Total Transport

MAY 31 - JUN 29, 1955
V. Integrated Total Transport

AUG 2 - AUG 31, 1955
V. Integrated Total Transport

AUG 30 - SEP 28, 1955
V. Integrated Total Transport

OCT 1 - OCT 30, 1955
V. Integrated Total Transport

NOV 1 - NOV 30, 1955
Section VI

Integrated Geostrophic Transport
VI. Integrated Geostrophic Transport

JAN 29 - FEB 27, 1955
VI. Integrated Geostrophic Transport

APR 30 - MAY 29, 1955
VI. Integrated Geostrophic Transport

MAY 31 - JUN 29, 1955
VI. Integrated Geostrophic Transport

JUL 2 - JUL 31, 1955
VI. Integrated Geostrophic Transport

AUG 2 - AUG 31, 1955
VI. Integrated Geostrophic Transport

NOV 1 - NOV 30, 1955
VI. Integrated Geostrophic Transport

NOV 29 - DEC 28, 1955