An Estimate of the Scale-length of Internal Waves in the Seasonal Thermocline near Ocean Station "P"

by

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I. Mechanics

The governing precept of the cruise plan was the intensive sampling, by bathythermograph, of a small area of the sea during a heating period. Existing equipment and methods required that a compromise be struck between spatial coverage, temporal coverage and sampling density. It was decided that an area of about 80 square miles could be adequately covered in eight hours by one ship - the ship travelling at seven to eight knots and bathythermograph casts being taken at five-minute intervals (every one-half mile on selected traverses). Because of the close spacing of the casts, Loran positioning was inadequate. Instead, the C.N.A.V. "Oshawa" determined position relative to the weathership, C.C.G.S. "St. Catharines", by radar. The "St. Catharines", in turn, stayed near an anchored buoy which was positioned at 50°N, 145°W (Station "P"). This procedure was used on July 19 and July 20, 1963 (Figs. 5 and 6). On the evening of July 20, the C.N.A.V. "Oshawa" departed for Esquimalt, B.C. On July 21, the C.C.G.S. "St. Catharines" carried out a limited bathythermograph survey by radar navigation with respect to the buoy (Fig. 7).

These data are shown in the Appendix.

II. Calculations and results

A. Thermal transients

Because there was little ocean heating during the survey, thermal transients were not found.
B. **Treatment of bathythermograms in order to obtain the internal wave topography**

The procedure was to approximate the trace in the neighbourhood of the upper part of the seasonal thermocline (hereafter, the upper thermocline) by straight lines and read the depths at the points of intersection of these straight lines (Fig. 1). In the few traces which were not well approximated at the bottom of the thermocline by this method, no depth was recorded.

![Diagram](image)

**Fig. 1.** Technique used for determining depths of limits of upper thermocline.

The contoured topographies of these depths (Figs. 2, 3, 4) give us approximate representations of the internal wave topography at the top and bottom of the upper thermocline.

C. **Randomness of the internal waves**

The internal wave topographies (Figs. 2, 3, 4) have the appearance of random seas.
D. **Estimate of scale-length of internal waves in the upper thermocline**

In order to obtain a scale length for the internal waves of significant amplitude, the arithmetic mean and the standard deviation of the depth of the feature being followed (e.g., the top or bottom of the upper thermocline) were calculated for each day. Then, on the topographic charts (Figs. 2, 3, 4) the average distance separating those neighbouring crests or troughs, whose depths differed from the mean by more than one standard deviation, were determined for each day. The results are presented in Table I. It is to be noted that, on July 19 and July 20, the sampling covered wavelengths between one and sixteen miles. On July 21, the sampling covered wavelengths between one and eight miles.

E. **The thickness of the upper thermocline**

The thickness of the upper thermocline was examined. There is no evidence here that the thickness of the upper thermocline is preserved. Rather, this thickness, during each day's survey, has a standard deviation near those of the depths of the top and bottom of the upper thermocline (Table II). This would imply that the variations in depth of the top and bottom of the upper thermocline are not in phase.

III. **Discussion**

A. **The technique for obtaining internal wave topography**

It is reasoned that following a feature of the temperature structure rather than an isotherm is the more fundamental approach.

B. **Errors in measurement**

An error, in depth, due to lag is inherent in the bathythermographs. Such an error is assumed to have been constant here and, therefore, to have had no effect on the variations of the depth of the thermocline which we are examining.
An error of within $\pm 1$ metre in depth will have been imparted to the bathythermograms by the motion of the ship in the waves which were about one metre in amplitude.

Errors in reading the depths of the limits of the thermocline from the bathythermograms are estimated to be within $\pm 1$ metre.

The total effective error of measurement is estimated to be within $\pm 2$ metres.

These errors are probably circumvented in the estimation of a scale length for internal waves in the neighbourhood of the thermocline because the total effective error of measurement is less than any of the calculated standard deviations which were chosen to define crests and troughs.

The thickness of the upper thermocline is a parameter which is free from errors due to the ship's motion.

C. Contours

In view of the size of the estimated error, a contour interval of one metre may seem unrealistic. This contour interval was used because the resulting density of contour lines helps one to imagine the three-dimensional topography.

The track charts followed on July 19 and July 20 yield non-uniform sampling densities (Figs. 2 and 3). I suggest that this situation leads to biased contouring. In Fig. 3, for example, there are about fourteen lines of data points across the field from north to south and five lines across the field from east to west. A crest or trough, aligned east-west, will be sampled at widely-spaced intervals compared to a crest or trough aligned north-south and, therefore, will be suppressed in the contouring. Thus the apparent north-south continuity in the figures may be an artifact caused by the suppression of crests aligned east-west. I note that the mean depths
along the east-west traverses for each day vary in a not irregular way (Table III). This seems to imply that there exist significant internal wave components, aligned east-west, which are not apparent in the contoured diagrams for July 19 and July 20.

The track chart followed on July 21 does yield a uniform sampling density. However, because the sampling pattern was essentially spiral, it is difficult to estimate the effect on the contoured data of the lengthy sampling period.

D. A suggested track chart

An optimum sampling pattern might incorporate features of both of the above patterns. Consider a pattern based on that of Fig. 7 but consisting of, say, a traverse with ten bathythermograms (1 each half-mile) followed by a return traverse of ten bathythermograms displaced one-half mile northward - and so on for ten traverses. This pattern would yield a uniform sampling density and preclude much of the difficulty of estimating the effect on the data of the lengthy sampling period.

IV. Conclusions

Thermal transients were not observed at Station "P" from July 19 to July 21, 1963 because strong heating conditions did not occur.

The topographies of the top and bottom of the upper thermocline at Station "P" from July 19 to July 21, 1963, have the appearance of random seas.

Topographies of the top and bottom of the upper thermocline for the same day do not seem to be related.

In the internal wave spectrum in the neighbourhood of the upper thermocline at Station "P" from July 19 to July 21, 1963, the three to ten kilometre wavelengths were significant in the sense that the average crest-to-crest separation of the larger 32% of the wave crests sampled was from three
kilometres at the bottom of the upper thermocline on July 21 to ten kilometres at the top of the upper thermocline on July 19.

The crack occurs following our July 21 data giving a number of ambiguous results.

However, because the sampling parcels are extensively dispersed it is difficult to estimate or suggest a crack date.

Table I. An estimate, $\bar{\lambda}_c$, of the scale-length of internal waves in the upper thermocline.

<table>
<thead>
<tr>
<th>Date</th>
<th>$\bar{D}_T$</th>
<th>$\bar{D}_B$</th>
<th>$\sigma$</th>
<th>$\bar{\lambda}_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 19, 1963</td>
<td>23.2</td>
<td>28.4</td>
<td>2.9</td>
<td>9.3 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>25.4</td>
<td>29.3</td>
<td>3.2</td>
<td>7.7 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>24.9</td>
<td>27.8</td>
<td>2.5</td>
<td>6.4 ± 0.8</td>
</tr>
</tbody>
</table>

$\bar{D}_T$ is the average depth, in metres, of the top of the upper thermocline.

$\bar{D}_B$ is the average depth, in metres, of the bottom of the upper thermocline.

$\sigma$ is the standard deviation, in metres, of the depth of the top/bottom of the upper thermocline.

$\bar{\lambda}_c$ is the average crest-to-crest separation, in kilometres, of the larger 32% of the wave crests.
Table II. Standard deviation, in metres, of the thickness of the upper thermocline.

<table>
<thead>
<tr>
<th>Date</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 19, 1963</td>
<td>3.35</td>
</tr>
<tr>
<td>July 20</td>
<td>2.38</td>
</tr>
<tr>
<td>July 21</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Table III. Mean depths, in metres, east-west traverses on July 19 and July 20

<table>
<thead>
<tr>
<th>Traverse</th>
<th>Mean depth top July 19</th>
<th>Mean depth bottom July 19</th>
<th>Mean depth top July 20</th>
<th>Mean depth bottom July 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most southerly</td>
<td>22.0</td>
<td>26.9</td>
<td>23.8</td>
<td>27.7</td>
</tr>
<tr>
<td>Second most southerly</td>
<td>22.5</td>
<td>27.2</td>
<td>27.0</td>
<td>30.7</td>
</tr>
<tr>
<td>Third most southerly</td>
<td>22.0</td>
<td>27.9</td>
<td>26.3</td>
<td>29.6</td>
</tr>
<tr>
<td>Fourth most southerly</td>
<td>24.7</td>
<td>29.8</td>
<td>25.5</td>
<td>28.9</td>
</tr>
<tr>
<td>Most northerly</td>
<td>25.9</td>
<td>34.0</td>
<td>23.9</td>
<td>29.4</td>
</tr>
</tbody>
</table>
Figure 4.
APPENDIX

Caution

Twenty minutes must be added to the times (GMT+10 h.20 m.) given for bathythermograms for July 19 and July 20, 1963, in order for these to coincide with times (GMT+10 h.) given on the track charts for these dates.
Figure 5. Track-chart showing bathythermograph stations (o) and corresponding times (GMT+10 h.). C.N.A.V. "Oshawa", July 19, 1963.
C.N.A.V. "Oshawa", Space-time series

BTgm from 63-07-19-0940 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-0945 to 63-07-19-1035 GMT+10h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-1040 to 63-07-19-1150 GMT+10 h. 20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-1153 to 63-07-19-1250 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

Btgs from 63-07-19-1255 to 63-07-19-1352 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-1358 to 63-07-19-1455 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

B T g m s from 63-07-19-1500 to 63-07-19-1605 GMT+10 h. 20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-1610 to 63-07-19-1705 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-19-1710 to 63-07-19-1820 GMT+10 h.20 m.
C.N.A.V. "Oshawa" Space-time series

BTgms from 63-07-19-1825 to 63-07-19-1840 and 1720, 1730 GMT+10 h.20 w.
Figure 6. Track-chart showing bathythermograph stations (o) and corresponding times (GMT+10 h.). C.N.A.V. "Oshawa", July 20, 1963.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-0847 to 63-07-20-0940 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-0945 to 63-07-20-1030 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1035 to 63-07-20-1130 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1135 to 63-07-20-1230 GMT±10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1235 to 63-07-20-1330 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1335 to 63-07-20-1430 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1435 to 63-07-20-1530 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1535 to 63-07-20-1630 GMT+10h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1635 to 63-07-20-1730 GMT+10 h.20 m.
C.N.A.V. "Oshawa", Space-time series

BTgms from 63-07-20-1735 to 63-07-20-1815 GMT+10h.20 m.
Figure 7. Track-chart showing bathythermograph stations (o) and corresponding times (GMT+10 h.). C.C.G.S. "St. Catharines", July 21, 1963.
C.C.G.S. "St. Catharines", Survey P-63-3, Space-time series

BTgms from 63-07-21-0820 to 63-07-21-0915 GMT+10

BTgms from 63-07-21-0920 to 63-07-21-1015 GMT+10
C.C.G.S. "St. Catharines", Survey P-63-3, Space-time series

BTgms from 63-07-21-1020 to 63-07-21-1115 GMT+10
C.C.G.S. "St. Catharines", Survey P-63-3, Space-time series

BTgms from 63-07-21-1235 to 63-07-21-1330 GMT+10
C.C.G.S. "St. Catharines," Survey P-63-3, Space-time series

BTgms from 63-07-21-1335 to 63-07-21-1430 GMT+10
E.C.G.S. "St. Catharines", Survey P-63-3, Space-time series

BTgms from 63-07-21-1535 to 63-07-21-1545 GMT+10