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DESIGN OF THE EASTERN BERING SEA TRAWL SURVEY:
ALLOCATION OF EFFORT AMONG STRATA

by

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INTRODUCTION

This report addresses the problem of selecting the magnitude and allocation of sampling effort among several strata to achieve reliable estimates of biomass for several demersal fish species in the eastern Bering Sea. Cochran (1963) presents a methodology for solving this problem for certain specific criteria as to the best allocation of sampling effort for the case of a stratified random sample.

For the situation where a single estimate is desired from the sample, the apportionment of effort which gives the minimum variance for the estimate is easily calculated. However, if several estimates are to be derived from the sampling, the problem is greatly complicated. The major difficulty is to define the appropriate criterion for the optimal allocation of sampling effort among strata. Any allocation that gives a minimum variance estimate of one variable will, in general, not give a minimum variance estimate for any other variable. Cochran suggests that a loss function be defined which expresses the cost associated with variance of the estimates. This loss function (L) will be a linear function of the variances:

$$L = \sum_{i=1}^{n} a_i \, V(\hat{B}_i) \qquad (1)$$

where:

$V(\hat{B}_i)$ = variance of the estimated biomass of the ith species
a_i = species specific coefficient
n = number of species for which biomass estimates are to be derived.

Intuitively, one would expect the magnitude of loss from a particular estimate to be proportional to its variance.

In addition to losses associated with highly variable estimates, there is the cost associated with carrying out the survey. This cost may be expressed by the following function:

$$C = c_o + \sum_{h=1}^{n_h} c_h E_h \tag{2}$$

where:

C = total cost of the survey

c_o = initial cost of the survey

c_h = cost of a single trawl haul in stratum h

E_h = total effort in stratum h

n_h = number of stratum

In Cochran's scheme, the best allocation of effort would be that which minimizes C + L.

It is straightforward to determine the nature of the cost function, which reflects the cost of sampling. It is not so clear how to express the losses in real dollars that result from poor estimates of demersal fish biomass. The magnitudes of C and L must be comparable since they are in the same units. The cost of sampling increases with increased sampling effort, whereas L decreases. The optimal allocation of effort is that which precisely balances C and L.

Determination of the coefficients of the loss function is difficult. The losses from a poor estimate of pollock would be presumably different from losses from a poor estimate of Alaska plaice. Pollock are very abundant and the mainstay of the Bering Sea fishery whereas Alaska plaice are not heavily exploited nor abundant, but these differences would be difficult to state precisely, particularly in view of future changes in species targeted on by the fishery.

In this report I have avoided some of these problems by framing the problem somewhat differently than Cochran. I suggest that the sampling be designed to satisfy some minimum tolerance expressed as the width of the confidence interval on the estimated biomass for each species. The best allocation of sampling effort would be that which satisfies these tolerance requirements and costs the least.

I might remind the reader that the problems alluded to in trying to precisely state the losses associated with variable estimates are not solved. Rather, they have been ignored and are implicit in the assumed vector of desired minimum tolerances.

THEORY

Expressions are derived for the optimal allocation of sampling effort for the three definitions of optimality alluded to in the previous section.

1. Sampling to obtain a single biomass estimate which minimizes variance for a fixed level of effort.

2. Sampling to obtain several biomass estimates that minimize a cost plus loss function.

3. Sampling to obtain several biomass estimates which costs the least and achieves a minimum tolerable width on the confidence interval for the estimated biomass for each species.

The following notation is used in the theory presented below:

B_j = biomass of the jth species
A_h = area of stratum h
E_h = number of units of sampling effort in stratum h
b = area swept by one standard haul
y_{jhi} = CPUE of species j in stratum h from haul i

$\overline{y}_{jh}$ = sample mean CPUE of species j in stratum h

n = true mean CPUE of species j in stratum h

σ_{jh}^{2} = true variance of the CPUE of species j in stratum h

The objective of the survey is to estimate $\hat{B}_j$. This estimate is given by

$$\hat{B}_j = \frac{1}{b} \sum_{h=1}^{n_h} A_n \overline{y}_{jh} \tag{3}$$

The variance of B_j is given by

$$V(\hat{B}_j) = \frac{1}{b^2} \sum_{h=1}^{n_h} \frac{A_n^2 \sigma_{jh}^2}{E_h} \tag{4}$$

Sampling to Obtain a Single Biomass Which Minimizes Variance for a Fixed Level of Effort

The problem here is to determine how a fixed amount of sampling effort is to be apportioned among strata to achieve the best estimate of biomass for a particular species j. More formally, we choose the set of E_h's to minimize $V(B_j)$ (eq. 4) subject to a constant that sampling costs (eq. 1) are fixed. This is equivalent to minimizing

$$U = V(B_j) + \lambda \left[\left(\sum_{h=1}^{n_h} c_h E_h \right) - C + c_o \right]$$

The set of E_h's that minimize U (see Cochran 1975, pp. 95-97 for the derivation) is given by the following equation:

$$\frac{E_h}{E} = \frac{A_h \sigma_{jh}/\sqrt{c_h}}{\sum_{h=1}^{n_h} A_h \sigma_{jh}/\sqrt{c_h}} \tag{5}$$

where E = total sampling effort

$$= \sum_{h=1}^{n_h} E_h$$

Note that the magnitude of sampling (E) is determined by the cost of sampling C.

Sampling to Obtain Several Biomass Estimates Which Minimize a Cost Plus Loss Function

To obtain the optimal allocation of sampling effort defined by this criterion, we need, a priori, a specified loss function (eq. 2) and a cost function (eq. 1). One wants to choose the set of E_h's to minimize C + L where

$$C + L = c_o + \sum_{h=1}^{n_h} c_h E_h + \sum_{j=1}^{n_j} a_j V(B_j)$$

$$+ \sum_h c_h E_h + \sum_j a_j \sum_h \frac{A_n^2 \sigma_{jh}^2}{b^2 E_h}$$

The E_h's which minimize C + L are those that satisfy

$$\frac{\partial (C + L)}{\partial E_h} = 0$$

for all h.

$$\frac{\partial (C + L)}{\partial E_h} = c_h - \frac{1}{b^2} \sum_j \frac{a_j A_h^2 \sigma_{jh}^2}{E_h}$$

Setting the derivatives equal to zero and solving for E_h, one gets

$$E_h = \frac{\left[\sum a_j A_h \sigma_{jh}^2\right]^{1/2}}{b\sqrt{c_h}} \qquad (6)$$

Sampling to Obtain Several Biomass Estimates to Achieve a Minimum Tolerable Width on the C.I. for B_j for Each Species j

Here we require the allocation of sampling effort to be sufficient for the variances to exceed some a priori tolerances, that is, the E_h's are chosen so that

$$V\,(B_j) \leq U_j \qquad (7)$$

for all j. It is convenient to express U_j as a fraction of the biomass:

$$U_j = u_j B_j$$

The problem is to find the least expensive allocation of effort that satisfies the above requirement. In the Bering Sea surveys, since some sampling is required in all strata and the cost per haul is the same in each stratum, the allocation of effort which is least expensive is the one that contains the lowest number of hauls.

I was not able to find a closed form expression for the E_h's (i.e., an analytical solution), but I was able to find the optimal allocation of effort by numerical methods. The algorithm that was used is outlined below.

Step 1. For each species, determine the optimal vector of effort allocation by stratum which satisfies the tolerance criterion (i.e., this is the minimum number of hauls which satisfies the criterion). The relative apportionment of effort among strata is given by 5 but we need the total level of effort:

$$E_j = \sum_h E_{jh}$$

where E_{jh} is the optimal allocation in stratum h. To find E_j observe that multiplying eq. 4 by E_j/E_j yields:

$$V(\hat{B}_j) = \frac{1}{E_j b^2} \sum_h \frac{A_h^2 \sigma_{jh}^2}{E_{jh}/E_j} \tag{8}$$

Now $V(\hat{B}_j)$ is minimized if

$$\frac{E_{jh}}{E_j} = \frac{A_h \sigma_{jh}}{\sum_h A_h \sigma_{jh}} \tag{9}$$

To find E_j which satisfies eq. 7, one substitutes eq. 9 into eq. 8 and substitutes this new expression for $V(\hat{B}_j)$ into eq. 7. Solving for E_j yields

$$E_j = \frac{1}{u_j^2} \frac{(\sum_h A_h \sigma_{jh})^2}{B_j^2 b^2} \tag{10}$$

Given E_j, the optimal vactor of effort allocation for the jth species ($\underline{E}^*(j) = \{E_{jh}\}$) is readily determined from eq. 9.

Step 2. Determine the species for which the minimum sample size necessary to meet tolerances is the greatest. Call this species j'. The optimal allocation of effort associated with species j' will be used as the starting point for the algorithm.

Call this $\underline{E}^*(j')$.

Step 3. Find the vector of variance of biomass for each species for the effort allocation $E^*(j')$.

Call this $\underline{V}$ $(\underline{E}^*(j'))$ = set of V_i (B_i) evaluated at $\underline{E}^*(j')$.

Step 4. The effort $\underline{E}^*(j')$ is a lower bound on the effort allocation needed to meet tolerances for all species. However, we can eliminate those species where the variance of biomass at $\underline{E}^*(j')$ meets tolerances from additional consideration. Thus, we eliminate those species where $V_i\ (\underline{E}^*(j')) \leq U_i$.

Call this reduced set of species S. Additional sampling is necessary to meet tolerances for any species contained in S.

Step 5. We must use an iterative procedure to increase sampling effort above that of $\underline{E}^*(j')$ until tolerances are met. To do this most efficiently, one wants to place the additional sampling effort in the stratum where the variance is reduced most quickly. To determine this, evaluate

$$\frac{\partial V_k}{\partial E_h} = \frac{1}{b^2} \frac{A_h^2 \sigma_{kh}^2}{E_h^2} \tag{11}$$

at the effort allocation $\underline{E}^*(j')$ for $h = 1, n_h$ and $k = 1, n_k$, where n_k equals the number of species in S. Now compute

$$D_h = \sum_{k=1}^{n_k} \frac{\partial V_k}{\partial E_h} \tag{12}$$

for each h. If we place additional sampling effort in the stratum for which D_h is the greatest, the tolerance will be met with the least amount of additional effort.

Call this stratum h*.

Step 6. Increase the sampling effort in stratum h* and recompute the vector of variances for the new effort allocation (E**).

Eliminate those species for which $V_i\ (E^{**}) \leq U_i$ from the set S.

Step 7. Check to see if there are species in set S. If S is empty, we go to step 8; otherwise, we must return to step 5 and increase the sampling effort in the appropriate stratum.

Step 8. Thus far in the algorithm, sampling effort has always been increased. Although the effort allocation reached at the initiation of step 8 meets the required tolerances, one has no assurance that this is the least costly allocation of effort.

To check this, the effort is systematically reduced until tolerances are just exceeded. The effort allocation just prior to this point is the optimal allocation of effort.

The effort allocation produced at step 7 is the starting point for this systematic reduction of effort. The effort in the stratum with minimum D_h is reduced by 1. The vector of variances is then computed. If tolerances are exceeded, the effort in that stratum is returned to that of the previous iteration. No further reduction of effort occurs again for that stratum. This process is continued until an effort reduction in all strata produces intolerable variance.

RESULTS

The application of the theory of the previous section to the design of the Bering Sea survey requires information on the underlying distribution for the catch of each species in each stratum. The 1975 and 1976 OCSEAP surveys of the Bering Sea provided this information. The raw catches were standardized to catch per unit effort by the following equation:

$$\text{CPUE} = \frac{C}{d\, f_{jv}}$$

where C = raw catch in pounds

d = distance towed in knots

f_{jv} = fishing power coefficient for species j and vessel v (Table IX-2, p. 168, in Pereyra, et al., 1976, and Table 2, p. 9, in Bakkala and Smith 1978).

The sampling strata for the 1975 survey (Fig. 1) and the 1976 survey (Fig. 2) were previously determined. This analysis assumes that stratum definitions are fixed and appropriate.

The sample mean CPUE ($\overline{X}$) and sample standard deviation (s) were determined by species and stratum for both the 1975 and the 1976 surveys (Table 1). These specified the underlying probability distribution of CPUE.

Based on the assumed underlying probability distribution of catch (Table 1), the optimal allocation of sampling effort for the four strata was computed (Table 2) for each year. To get an idea of how much the variance can be reduced by judicious allocation of sampling effort, the variances realized in the actual allocation of sampling effort used in the 1975 and 1976 surveys were compared with the minimum possible variance with optimal allocation of effort for each species (Tables 3, 4). The realized variance was, in the case of king crab during 1975, 300% greater than the minimum variance achievable with 525 hauls. Table 4 also illustrates the simultaneous sampling problem. If the distribution of species among strata differs by species, any designed effort allocation will be highly suboptimal for some species.

No attempt was made to compute the optimal effort allocation for the situation where one is trying to minimize a cost plus loss function. After determining the loss function, it is a simple matter to compute the optimal allocation of effort (eq. 6).

A computer program is available (Appendix 1) which computes the least costly effort allocation to obtain an a priori set of minimum tolerances for each species. The computations are based on the underlying probability distribution of catch by stratum and species (Table 1).

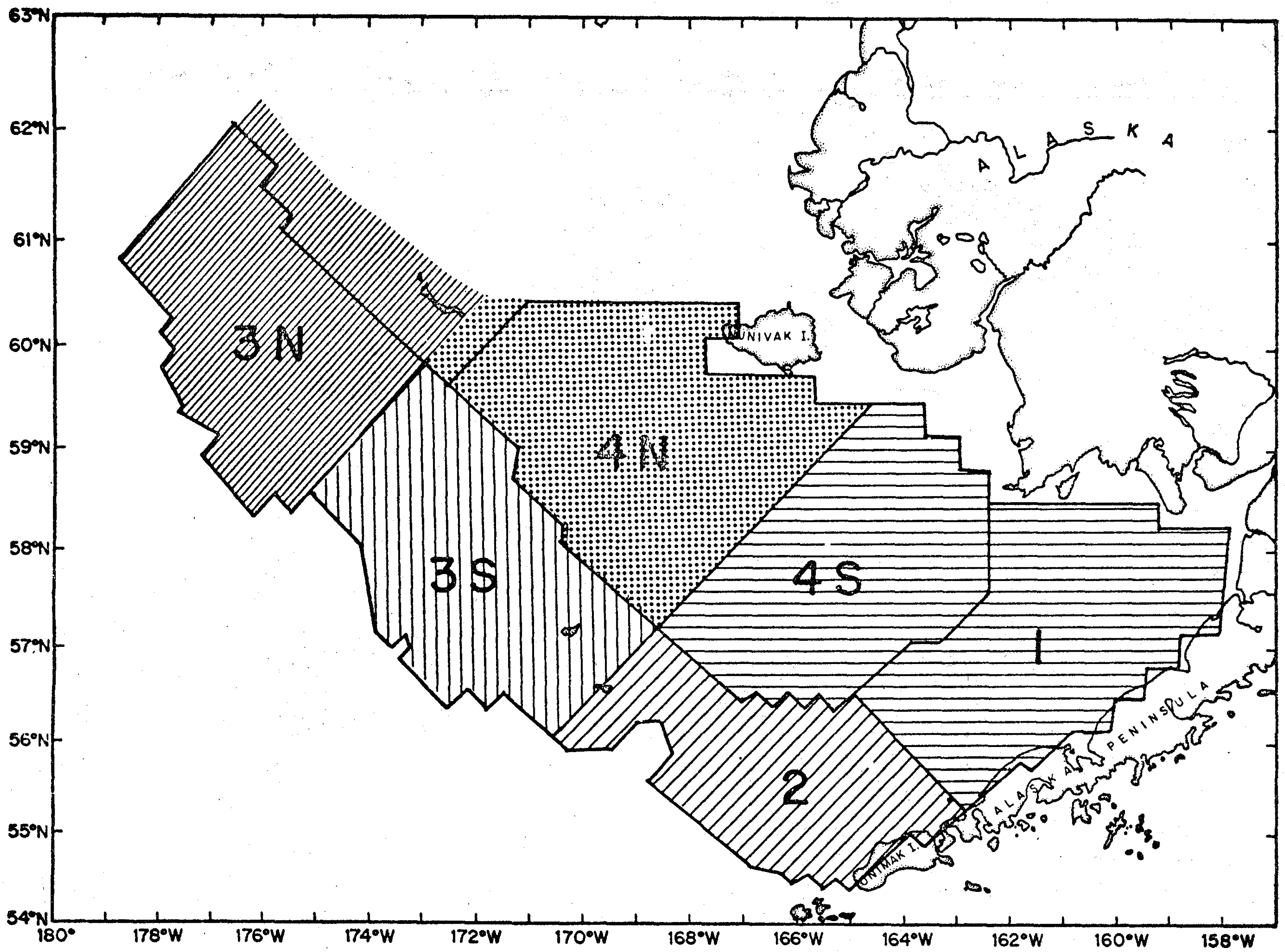

Fig. 1. Strata definitions for the 1975 survey.

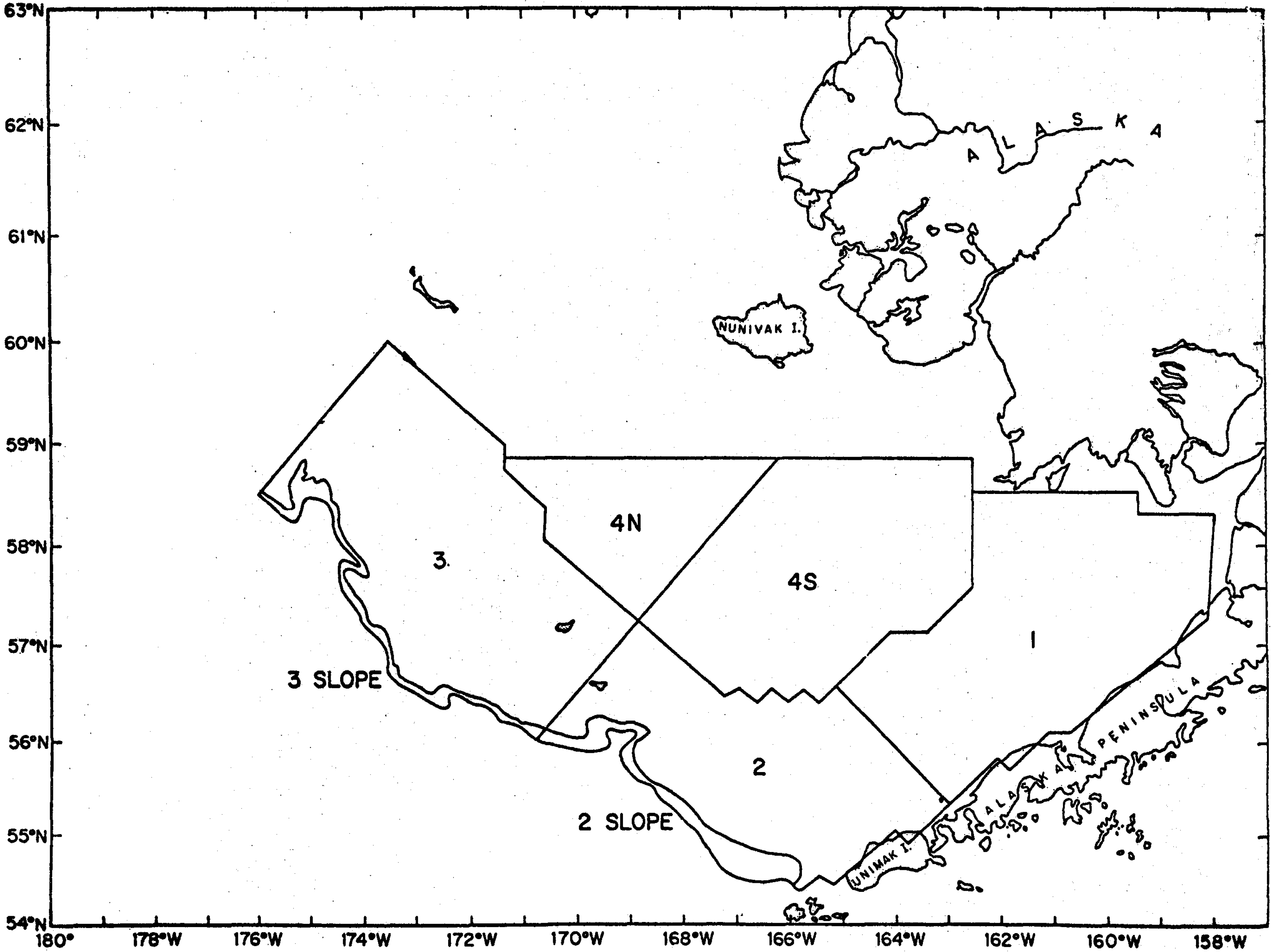

Fig. 2. Strata subdivisions of the study area used for the 1976 spring trawl survey.

Table 1. Sample mean ($\overline{X}$) and standard deviation (s) of CPUE by species and stratum for the 1975 and 1976 surveys.

	1975			1976		
Species	Stratum	$\overline{X}$	s	Stratum	$\overline{X}$	s
Pollock	1	43.59674	120.26052	1	11.33097	31.26435
	2	926.51950	1458.12230	2	487.09981	822.34687
				2SL	369.85076	817.80223
	3S	256.23587	487.01716	3	149.27592	566.32034
	3N	821.39062	772.21037			
				3SL	62.96037	110.05414
	4S	162.37259	536.87748	4S	5.20422	16.89372
	4N	2.31464	6.49782	4N	3.77022	10.18429
Yellowfin sole	1	423.37107	501.94993	1	1057.93688	3462.47257
	2	61.59947	179.52462	2	518.95586	2128.79031
				2SL	0	0
	3S	4.97630	16.22015	3	88.53870	231.43986
	3N	0.00942	0.06456			
				3SL	0.55944	1.85545
	4S	264.29289	292.39679	4S	194.04834	311.54179
	4N	77.34312	161.00155	4N	97.81182	128.12755
Pacific cod	1	3.49634	6.90308	1	0.82431	2.87217
	2	29.06494	43.98140	2	70.32219	150.18451
				2SL	111.87883	134.17130
	3S	8.40544	21.67160	3	18.73287	57.70996
	3N	15.09508	28.75562			
				3SL	68.29725	106.66482
	4S	2.03179	5.54870	4S	0.50495	1.34826
	4N	0.16662	0.94304	4N	0.09052	0.12072
Rock sole	1	52.73122	77.45648	1	21.43193	61.82267
	2	42.46425	85.94491	2	121.40953	264.68862
				2SL	6.19284	21.01829
	3S	14.52097	45.83858	3	10.86037	26.33076
	3N	0.18571	0.65159			
				3SL	1.06409	1.55016
	4S	26.13725	40.13327	4S	72.39328	510.25494
	4N	5.32991	10.32564	4N	1.57136	7.21512
Flathead sole	1	6.50988	12.73520	1	1.47475	4.51391
	2	60.70975	75.87877	2	91.27356	182.14670
				2SL	28.82636	55.31995
	3S	14.77143	19.49365	3	8.91561	18.51311
	3N	15.54544	24.15255			
				3SL	9.93887	8.87540
	4S	5.48854	11.27942	4S	0.33627	1.26703
	4N	0.02075	0.12265	4N	0.28196	0.6355

Table 1, cont'd

	1975			1976		
Species	Stratum	$\bar{X}$	s	Stratum	$\bar{X}$	s
Alaska plaice	1	15.26098	32.26444	1	43.75730	68.67960
	2	4.79036	13.55667	2	40.93581	108.35376
				2SL	0	0
	3S	0.22146	0.74931	3	7.83826	18.51143
	3N	0.03096	0.20758			
				3SL	0	0
	4S	51.32180	60.56912	4S	118.74271	156.24923
	4N	29.54279	43.38017	4N	42.47189	70.28102
Arrowtooth flounder	1	0.27286	1.04887	1	0	0
	2	20.01730	20.74427	2	21.60141	27.78325
				2SL	159.24382	274.91482
	3S	4.4826	10.78425	3	2.67458	10.82537
	3N	0.26247	1.52508			
				3SL	28.70503	24.05569
	4S	0.05481	0.33535	4S	0	0
	4N	0	0	4N	0	0
Greenland halibut	1	3.23697	5.21522	1	1.50962	5.72817
	2	9.38971	22.01444	2	16.73074	20.14990
				2SL	80.82332	123.78494
	3S	26.33911	35.40264	3	19.70810	24.03481
	3N	69.35185	51.48867			
				3SL	19.45541	16.91898
	4S	10.64927	17.27896	4S	0.8464	2.90198
	4N	3.50725	4.92612	4N	1.0172	3.64587
Red king crab	1	85.58068	338.41331	1	73.98704	153.08087
	2	40.57193	157.65148	2	34.44504	76.18001
				2SL	0	0
	3S	0.15542	1.22646	3	0.25232	1.72255
	3N	0	0			
				3SL	0	0
	4S	1.78083	5.94130	4S	1.10905	2.64610
	4N	0.06312	0.23542	4N	0.04383	0.20557
C. bairdi	1	18.25223	51.05186	1	54.45615	225.61809
	2	52.52149	85.25828	2	65.16156	87.79544
				2SL	4.64547	13.50969
	3S	21.34113	63.89220	3	16.54014	30.59012
	3N	7.33114	28.57692			
				3SL	4.80882	5.04353
	4S	2.57568	7.17763	4S	1.89676	6.46668
	4N	0.90523	5.45284	4N	0.43917	2.01229

Table 1, cont'd

	1975			1976		
Species	Stratum	$\bar{X}$	s	Stratum	$\bar{X}$	s
C. opilio	1	26.57357	68.79920	1	24.23715	68.01590
	2	14.18295	20.72095	2	118.81098	308.96532
				2SL	5.07388	14.51212
	3S	94.36835	215.91822	3	107.18312	139.55828
	3N	40.49587	45.06002			
				3SL	168.29197	238.60899
	4S	60.66815	103.49332	4S	166.40242	360.34515
	4N	38.63199	65.40740	4N	295.43680	374.34057

Table 2. Optimal allocation of a fixed amount of effort among strata to achieve minimum variance estimate for the biomass of the respective species (%) as well as level of effort necessary for the standard error to be 10% of the mean balance.

		1	2	2SL	3S	3N	3SL	4S	4N	Effort level
Pollock	1975	4.0	42.0		15.0	21.8		17.0	0.2	241
	1976	2.4	48.2	5.5	42.2		0.3	1.1	0.3	525
Yellowfin sole	1975	44.8	13.8		1.3	0.0		24.7	15.4	190
	1976	61.5	28.8	0.1	4.0		0.0	4.9	0.9	1,039
Pacific cod	1975	7.2	39.8		20.9	25.5		5.5	1.1	347
	1976	1.5	60.1	6.2	29.4		2.2	0.6	0.0	496
Rock sole	1975	31.8	30.5		17.3	0.2		15.6	4.6	333
	1976	8.3	27.1	0.3	3.4		0.0	60.5	0.4	1,540
Flathead sole	1975	10.0	51.4		14.1	16.1		8.4	0.1	197
	1976	2.7	82.7	2.9	10.7		0.2	0.7	0.1	409
Alaska plaice	1975	21.7	7.9		0.5	0.1		38.6	31.3	222
	1976	20.6	24.8	0.4	5.4		0.4	41.3	8.0	269
Arrowtooth flounder	1975	3.4	58.7		32.6	4.2		1.0	0.5	193
	1976	0.4	37.4	42.4	18.5		1.7	0.4	0.4	276
Greenland halibut	1975	4.3	15.6		26.7	35.7		13.4	4.3	127
	1976	9.6	25.7	18.1	39.0		1.1	4.3	2.3	200
Red king crab	1975	70.3	28.2		0.2	0.1		1.2	0.1	1,545
	1976	71.1	27.0	0.2	0.8		0.2	1.1	0.1	450
C. bairdi	1975	23.2	33.4		26.7	11.0		3.1	2.7	563
	1976	68.4	20.3	0.4	9.0		0.1	1.7	0.2	725
C. opilio	1975	14.0	3.6		40.4	7.8		19.9	14.3	361
	1976	7.5	25.9	0.1	14.9		1.0	35.0	15.6	352

Table 3. Comparison of expected variance for the optimal allocation of effort to the observed variance for the actual allocation of effort for the 1975 survey.

Species		Effort per stratum						% Reduction in variance
		1	2	3S	3N	4S	4N	
Pollock	Actual	107	92	114	89	57	66	60.0
	Optimal	21	221	79	115	89	1	
Yellowfin sole	Actual	107	92	114	89	57	66	54.2
	Optimal	236	73	7	0	130	81	
Pacific cod	Actual	107	92	114	89	57	66	64.7
	Optimal	38	209	110	134	29	6	
Rock sole	Actual	107	92	114	89	57	66	71.0
	Optimal	167	160	91	1	82	24	
Flathead sole	Actual	107	92	114	89	57	66	53.6
	Optimal	53	270	74	84	44	1	
Alaska plaice	Actual	107	92	114	89	57	66	41.3
	Optimal	114	41	2	1	203	65	
Arrowtooth flounder	Actual	107	92	114	89	57	66	40.4
	Optimal	18	309	171	22	5	0	
Greenland halibut	Actual	107	92	114	89	57	66	70.8
	Optimal	22	82	140	188	71	23	
Red king crab	Actual	107	92	114	89	57	66	34.64
	Optimal	370	149	1	0	6	0	
C. bairdi	Actual	107	92	114	89	57	66	76.0
	Optimal	122	176	140	58	16	14	
C. opilio	Actual	107	92	114	89	57	66	70.3
	Optimal	74	19	213	41	105	75	

Table 4. Comparison of expected variance for the optimal allocation of effort to the observed variance for the actual allocation of effort for the 1976 survey.

		Effort per stratum							
Species		1	2	2SL	3	3SL	4S	4N	% Reduction in variance
Pollock	Actual	100	89	40	117	11	56	22	54.5
	Optimal	10	209	24	184	1	5	1	
Yellowfin sole	Actual	100	89	40	117	11	56	22	48.2
	Optimal	267	125	0	17	0	21	4	
Pacific cod	Actual	100	89	40	117	11	56	22	46.6
	Optimal	7	261	27	128	10	3	0	
Rock sole	Actual	100	89	40	117	11	56	22	30.9
	Optimal	36	118	1	15	0	263	2	
Flathead sole	Actual	100	89	40	117	11	56	22	29.4
	Optimal	12	360	13	47	1	3	1	
Alaska plaice	Actual	100	89	40	117	11	56	22	51.41
	Optimal	90	108	0	23	0	180	35	
Arrowtooth flounder	Actual	100	89	40	117	11	56	22	36.0
	Optimal	0	163	184	81	7	0	0	
Greenland halibut	Actual	100	89	40	117	11	56	22	76.3
	Optimal	42	112	79	170	5	19	10	
Red king crab	Actual	100	89	40	117	11	56	22	39.1
	Optimal	309	117	0	3	0	5	0	
C. bairdi	Actual	100	89	40	117	11	56	22	44.11
	Optimal	297	88	2	39	0	8	1	
C. opilio	Actual	100	89	40	117	11	56	22	53.55
	Optimal	33	113	1	65	5	152	68	

Results for some arbitrary vectors of tolerances are presented in Tables 5 and 6. Note that as tolerances decrease, the necessary total sample size increases rapidly (Fig. 3).

I am not able to offer an analytical proof that the effort allocation derived by the program is optimal. However, analysis of the algorithm's behavior strongly suggests that the resulting effort allocation is optimal. First, the same results occur for many different starting points other than the $\underline{E}^*(j)$ where total effort is greatest. Furthermore, the solution is a local minimum because decreasing the effort in any stratum results in intolerable variances.

As a further test of the optimality of the derived effort allocation, the program was used to predict the best effort allocation to achieve the tolerances realized in the 1975 and 1976 surveys. The results are shown on the lines with double asterisks in Tables 5 and 6. The predicted allocation should agree with the actual allocations if the algorithm is working properly. Unfortunately, the predicted effort allocations are greater than the actual effort allocations in both years (555 versus 525 in 1975, 448 versus 435 in 1976). In both cases the predicted proportion of effort allocated by stratum is very close to the observed proportion. This result is troublesome and suggests that the solution reached by the algorithm is not a global minimum. Further work would be needed to improve the algorithm. The predicted optimal effort allocations are very close (within 5%) and this work would be unnecessary fine-tuning, particularly in view of the yearly variation in the distribution of fishes by stratum in the Bering Sea. Using past data to specify the expected distribution of catch per unit effort in future surveys is approximate.

Table 5. Optimal allocation of sampling effort for various tolerance levels, based on 1975 survey.

Desired tolerance for each species	Stratum						
	1	2	3S	3N	4S	4N	Total
0.05	4,345	1,745	442	127	252	187	7,078
(%)	(61)	(25)	(6)	(2)	(4)	(3)	
0.07	2,183	890	223	65	129	96	3,586
0.10	1,070	436	110	32	63	47	1,758
0.15	476	194	50	14	28	21	783
0.20	268	109	28	8	16	12	440
0.25	172	70	18	6	10	8	283
*	1,077	436	70	38	26	9	1,656
**	105	132	118	81	67	52	555

*Optimal effort allocation for the following vector of tolerances: Pollock--0.1, yellowfin sole--0.1, Pacific cod--0.1, rock sole--0.2, flathead sole--0.2, Alaska plaice--0.2, arrowtooth flounder--0.2, Greenland halibut--0.2, red king crab--0.1, *C. bairdi*--0.1, *C. opilio*--0.2.

**Actual tolerances realized by the 1975 survey: Pollock--0.0873, yellowfin sole--0.0816, Pacific cod--0.1010, rock sole--0.0944, flathead sole--0.0835, Alaska plaice--0.1010, arrowtooth flounder--0.0953, Greenland halibut--0.585, red king crab--0.2912, *C. bairdi*--0.1187, *C. opilio*--0.0988.

Table 6. Optimal allocation of sampling effort for various tolerances, based on the 1976 survey.

Desired tolerance for each species	Stratum 1	2	2SL	3	3SL	4S	4N	Total
0.05	2,050	1,671	244	592	11	3,249	49	7,865
(%)	(26)	(21)	(3)	(8)	(11)	(41)		
0.07	1,046	852	125	305	5	1,901	26	4,260
0.10	513	418	61	145	3	812	13	1,965
0.15	227	186	27	86	2	361	6	874
0.20	127	104	16	35	1	204	3	491
0.25	81	67	10	23	1	130	2	314
*	579	299	23	179	3	158	9	1,250
**	101	89	46	126	4	57	22	448

*Optimal effort allocation for the following vector of tolerances: Pollock--0.1, yellowfin sole--0.1, Pacific cod--0.1, rock sole--0.2, flathead sole--0.2, Alaska plaice--0.2, arrowtooth flounder--0.2, Greenland halibut--0.2, red king crab--0.1, *C. bairdi*--0.1, *C. opilio*--0.2.

**Actual tolerances realized by the 1976 survey: Pollock--0.1487, yellowfin sole--0.2227, Pacific cod--0.1565, rock sole--0.3385, flathead sole--0.1787, Alaska plaice--0.1097, arrowtooth flounder--0.1327, Greenland halibut--0.0776, red king crab--0.1626, *C. bairdi*--0.1944, *C. opilio*--0.1230.

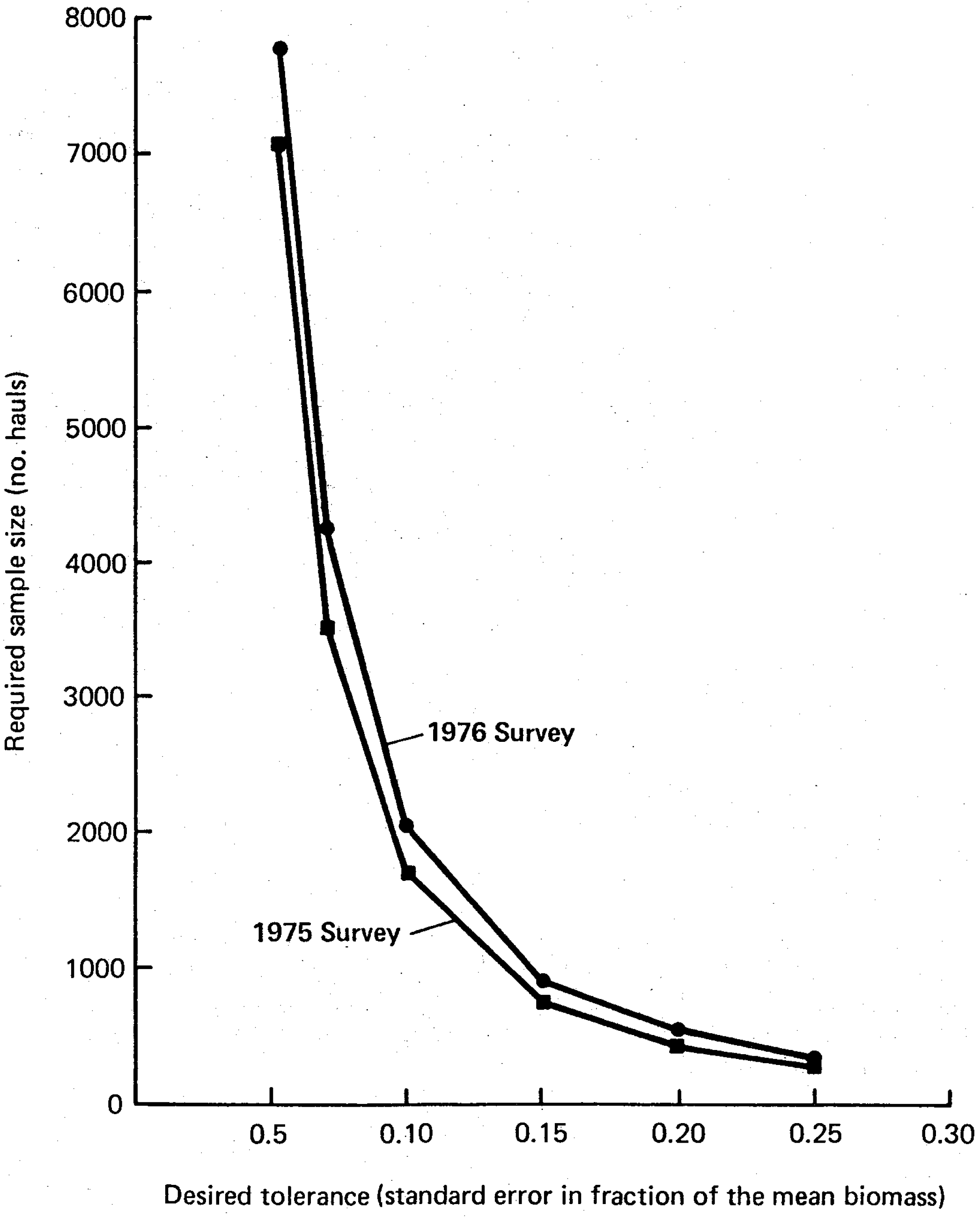

Figure 3.--Relation between desired tolerance and required sample size based on 1975 and 1976 survey data.

At this stage in the analysis, there appear to be two major discrepancies in the results based on the 1975 and 1976 surveys. The required sample sizes based on the 1976 survey are roughly 1.25 times greater than those required based on the 1975 survey (Fig. 3). Secondly, the allocation of sampling effort based on the 1976 survey requires that 41% of the effort be in stratum 4S, whereas the allocation based on the 1975 survey requires that only 4% be in stratum 4S. These discrepancies are due to one very large catch of rock sole in area 4S in 1976. This resulted in extreme variance of rock sole CPUE in area 4S (Table 1).

The results are sensitive to one or two species. Equation 10 can be used to assess which species the results are going to be most sensitive to. This is accomplished by ranking the species according to $E_j u_j^2$, where E_j is total effort required to meet tolerance u_j if the survey is used only to estimate biomass of species j. The results will be most sensitive to that species which has the highest $E_j u_j^2$. The results of the 1975 survey were most sensitive to king crab (Table 7) and the results of the 1976 survey were most sensitive to rock sole. The high effort required to meet a uniform tolerance of 0.1 for each species was due to 1,545 hauls necessary to achieve that for king crab alone based on the 1975 survey, and 1,540 hauls necessary to achieve that for rock sole alone based on the 1976 survey.

One means of circumventing this is to consider only a few of the most important species. I restricted the species considered to pollock, yellowfin sole, king crab, and tanner crab (<u>C. bairdi</u>). The results based on the 1975 and 1976 surveys (Table 8) are much more consistent with the reduced number of species. The relative allocations of effort among strata for the two years are very close. The magnitude of effort

Table 7. Values of $E_j u_j^2$ for each species based on 1975 and 1976 surveys.

1975				1976			
Species	$E_j U_j^2$	Relative value	Rank	Species	$E_j U_j^2$	Relative value	Rank
King crab	15.45	100	1	Rock sole	15.40	100	1
C. bairdi	5.63	36.4	2	Yellowfin sole	10.39	67.5	2
C. opilio	3.61	23.4	3	*C. bairdi*	7.25	47.1	3
Pacific cod	3.47	22.5	4	Pollock	5.25	34.1	4
Rock sole	3.33	21.6	5	Pacific cod	4.96	32.2	5
Pollock	2.41	15.6	6	King crab	4.50	29.2	6
Alaska plaice	2.22	14.4	7	Flathead sole	4.09	26.6	7
Flathead sole	1.97	12.8	8	*C. opilio*	3.52	22.9	8
Arrowtooth flounder	1.93	12.5	9	Arrowtooth flounder	2.76	17.9	9
Yellowfin sole	1.90	12.3	10	Alaska plaice	2.69	17.5	10
Greenland halibut	1.27	8.2	11	Greenland halibut	2.0	13.0	11

Table 8. Optimal allocation of sampling effort considering only pollock, yellowfin sole, king crab, and tanner crab (C. *bairdi*). Tolerances are uniformly equal to 0.1.

		Stratum								
		1	2	2SL	3S	3N	3SL	4S	4N	Total
1975	Effort level	1,076	436		75	31		27	9	1,654
	%	65.1	26.4		4.5	1.9		1.6	0.5	
1976	Effort level	609	299	24	179		2	51	9	1,173
	%	51.9	25.5	2.0	15.3		0.2	4.3	0.8	

required to meet tolerances based on the 1975 survey is greater than that required based on the 1976 survey. This was due to the high $E_j u_j^2$ observed for king crab based on the 1975 survey (Table 7). This was caused by much higher variance of king crab in 1975.

There are two ways to reduce the variance of king crab which would improve the surveys. First, instead of using the survey to estimate total biomass of crabs, one could use the survey to estimate total numbers of biomass of legal males. Catches of males are less variable because males are less aggregated than females and immature crabs (Dave Somerton, NORFISH, personal communication).

The second way to reduce this variation would be to redefine strata 1 and 2 into king crab and non-king crab areas. Large catches of king crab occur in a well-defined area. Zero catches of king crab consistently occur in large areas of strata 1 and 2. These zero catches greatly increase the estimated variance of CPUE.

Another problem is the inconsistent assignment of stratum weights during 1975 and 1976 (Table 9). The different surface areas of stratum used for the 1975 and 1976 biomass estimates would yield different allocations of effort even if the expected catches by species and stratum were the same.

Table 9. Surface areas (km^2) of sampling strata used for the 1975 and 1976 surveys.

Stratum	1975	%	1976	%
1	87,291	17.7	82,100	24.3
2	75,253	15.3	62,550	18.3
2SL			7,160	2.1
3S	80,336	16.3	79.570	23.5
3N	73,894	16.8		
3SL			3,240	1.0
4S	82,623	15.0	30,960	9.2
4N	93,598	19.0	72,350	21.4
Total	492,995		337,930	

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